GAUSS RECOVERY EPHEMERIS FOR CERES FOR DATES IN NOVEMBER-DECEMBER 1801

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This worksheet was constructed to convert Gauss's recovery ephemeris of Ceres [1] from geocentric ecliptic longitudes and latitudes to right ascensions and declinations.

Define needed constants and set the Mathcad worksheet ORIGIN to 1 so that subscripts start at unity rather than at zero.

$$DegPerRad := \frac{180}{\pi}$$

(Just need conversion from radians to degrees in this worksheet.)

$$\mathbf{ORIGIN} \equiv 1$$

1. Input ephemeris points as predicted by Gauss in 1801 December. The angles are geocentric ecliptic longitude and latitude. But the longitudes are expressed using the Zodiac Number Z*, where Z ranges from 0 to 11, then degrees and minutes. The geocentric ecliptic latitudes are just degrees and minutes.

(*To convert the ecliptic longitudes to ordinary degrees, multiply Z by 30 and then add to the degrees and minutes fields immediately following Z.)

$$Data := \text{READPRN}\left(\text{“Gauss\_Ceres\_MC\_4\_LVII\_p.647.txt”}\right)$$

$$Data = \begin{bmatrix} 1801 & 11 & 25 & 5 & 20 & 16 & 9 & 25 \\ 1801 & 12 & 1 & 5 & 22 & 15 & 9 & 48 \\ 1801 & 12 & 7 & 5 & 24 & 7 & 10 & 12 \\ 1801 & 12 & 13 & 5 & 25 & 51 & 10 & 37 \\ 1801 & 12 & 19 & 5 & 27 & 27 & 11 & 4 \\ 1801 & 12 & 25 & 5 & 28 & 53 & 11 & 32 \\ 1801 & 12 & 31 & 6 & 0 & 10 & 12 & 1 \end{bmatrix}$$

The data are year, month, day, geocentric ecliptic longitude (Z, deg, min), and geocentric ecliptic latitude (deg, min).

$$n := \text{rows}\left(Data\right) \qquad n = 7$$

2. Convert Gregorian dates to Julian dates. Convert geocentric ecliptic longitudes and latitudes to radians.

$$DayCount := \begin{bmatrix} 0 & 31 & 59 & 90 & 120 & 151 & 181 & 212 & 243 & 273 & 304 & 334 \end{bmatrix}^{\mathrm{T}}$$

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JED18(Year, Month, Day) := | JED ← 2378495.5
                           | for Y ∈ 1800 .. Year
                           |   if Y < Year
                           |     if mod(Y, 4) ≠ 0
                           |       JED ← JED + 365
                           |     else
                           |       if mod(Y, 100) = 0
                           |         if mod(Y, 400) = 0
                           |           JED ← JED + 366
                           |         else
                           |           JED ← JED + 365
                           |       else
                           |         JED ← JED + 366
                           |   else
                           |     JED ← JED + DayCount_Month + Day
                           |     if mod(Y, 4) ≠ 0
                           |       JED ← JED + 0
                           |     else
                           |       if Month > 2
                           |         if mod(Y, 100) = 0
                           |           if mod(Y, 400) = 0
                           |             JED ← JED + 1
                           |           else
                           |             JED ← JED + 0
                           |         else
                           |           JED ← JED + 1
                           |       else
                           |         JED ← JED + 0
                           | JED

JDTCalc(k) := | for i ∈ 1 .. k
              |   JDT_i ← JED18(Data_{i,1}, Data_{i,2}, Data_{i,3})
              | JDT
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$JDT := JDTCalc(n)$

$$JDT = \begin{bmatrix} 2379189.5 \\ 2379195.5 \\ 2379201.5 \\ 2379207.5 \\ 2379213.5 \\ 2379219.5 \\ 2379225.5 \end{bmatrix}$$

$$EcLng(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| Eclng_i \leftarrow \dfrac{\left(Data_{i,4} \cdot 30 + Data_{i,5} + \dfrac{Data_{i,6}}{60} \right)}{DegPerRad} \right. \\ Eclng \end{array} \right.$$

$$Lambda := EcLng(n)$$

Geocentric ecliptic longitudes are in radians.

$$EcLat(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| \begin{array}{l} EcLat_i \leftarrow \dfrac{\left| Data_{i,7} \right| + \dfrac{Data_{i,8}}{60}}{DegPerRad} \\ \text{if } Data_{i,7} < 0 \\ \quad \| ELDEC_i \leftarrow -ELDEC_i \end{array} \right. \\ EcLat \end{array} \right.$$

$$Beta := EcLat(n)$$

Geocentric ecliptic latitudes are in radians.

3. Transform geocentric ecliptic longitudes and latitudes to geocentric equatorial right ascensions and declinations, as per the equations in [2].

To do this, need oblquity of ecliptic as a function of Julian date. That is obtained from the following function which was taken from [3].

$$eps(JDT) := \frac{23.4392794444 - 0.01301021361 \cdot \dfrac{(JDT - 2451545.0)}{36525.0}}{DegPerRad}$$

$$RADec(JDT, Lambda, Beta) := \left\| \begin{array}{l} n \leftarrow \text{rows}(Lambda) \\ \text{for } i \in 1..n \\ \quad \left\| \begin{array}{l} \varepsilon \leftarrow eps(JDT_i) \\ Dec_i \leftarrow \text{asin}\left(\sin(Beta_i)\cdot\cos(\varepsilon) + \cos(Beta_i)\cdot\sin(\varepsilon)\cdot\sin(Lambda_i)\right) \\ x \leftarrow \cos(Beta_i)\cdot\cos(Lambda_i) \\ y \leftarrow -\sin(Beta_i)\cdot\sin(\varepsilon) + \cos(Beta_i)\cdot\cos(\varepsilon)\cdot\sin(Lambda_i) \\ RA_i \leftarrow \dfrac{\text{angle}(x, y)}{15} \end{array} \right. \\ \text{augment}(\text{augment}(JDT, RA \cdot DegPerRad), Dec \cdot DegPerRad) \end{array} \right.$$

$$RADEC := RADec(JDT, Lambda, Beta)$$

$$RADEC = \begin{bmatrix} 2379189.5 & 11.65584778 & 12.503225 \\ 2379195.5 & 11.78850405 & 12.06647497 \\ 2379201.5 & 11.91407314 & 11.68968616 \\ 2379207.5 & 12.03161694 & 11.38053293 \\ 2379213.5 & 12.14166928 & 11.15499668 \\ 2379219.5 & 12.24180545 & 11.01156347 \\ 2379225.5 & 12.33310162 & 10.94380594 \end{bmatrix}$$

First column is Julian date.
Second column is right ascension.
Third column is declination.

$$Date := \text{augment}\left(\text{augment}\left(Data^{\langle 1 \rangle}, Data^{\langle 2 \rangle}\right), Data^{\langle 3 \rangle}\right)$$

$$Array := \text{augment}\left(\text{augment}\left(Date, RADEC^{\langle 2 \rangle}\right), RADEC^{\langle 3 \rangle}\right)$$

$$Array = \begin{bmatrix} 1801 & 11 & 25 & 11.65584778 & 12.503225 \\ 1801 & 12 & 1 & 11.78850405 & 12.06647497 \\ 1801 & 12 & 7 & 11.91407314 & 11.68968616 \\ 1801 & 12 & 13 & 12.03161694 & 11.38053293 \\ 1801 & 12 & 19 & 12.14166928 & 11.15499668 \\ 1801 & 12 & 25 & 12.24180545 & 11.01156347 \\ 1801 & 12 & 31 & 12.33310162 & 10.94380594 \end{bmatrix}$$

This matrix provides Gregorian dates, right ascensions in hours, and declinations in degrees for Gauss's Ceres predictions for the final days of 1801. Note that von Zach recovered Ceres on 1801 December 31, but had previously, unknowingly observed it on 1801 December 7.

Write out Gauss's predictions for plotting on a star chart.

$$AlphaDelta := \text{augment}\left(Array^{\langle 4 \rangle}, Array^{\langle 5 \rangle}\right)$$

$$AlphaDelta = \begin{bmatrix} 11.6558 & 12.5032 \\ 11.7885 & 12.0665 \\ 11.9141 & 11.6897 \\ 12.0316 & 11.3805 \\ 12.1417 & 11.155 \\ 12.2418 & 11.0116 \\ 12.3331 & 10.9438 \end{bmatrix}$$

$$\text{WRITEPRN}(\text{"GAUSS.prn"}, AlphaDelta) = \begin{bmatrix} 11.65584778 & 12.503225 \\ 11.78850405 & 12.06647497 \\ 11.91407314 & 11.68968616 \\ 12.03161694 & 11.38053293 \\ 12.14166928 & 11.15499668 \\ 12.24180545 & 11.01156347 \\ 12.33310162 & 10.94380594 \end{bmatrix}$$

Write out Gauss's recovery ephemeris for input to Hd1_Hdc.

$$\text{WRITEPRN}(\text{"GAUSS\_RECOVERY\_EPHEMERIS.prn"}, Array) = \begin{bmatrix} 1801 & 11 & 25 & 11.65584778 & 12.503225 \\ 1801 & 12 & 1 & 11.78850405 & 12.06647497 \\ 1801 & 12 & 7 & 11.91407314 & 11.68968616 \\ 1801 & 12 & 13 & 12.03161694 & 11.38053293 \\ 1801 & 12 & 19 & 12.14166928 & 11.15499668 \\ 1801 & 12 & 25 & 12.24180545 & 11.01156347 \\ 1801 & 12 & 31 & 12.33310162 & 10.94380594 \end{bmatrix}$$

REFERENCES

[1] von Zach, F. X., *Monatliche Correspondenz* for 1801 December (Vol. 4, Article LVII, pp. 638-649). See table on p. 647.

[2] W. M. Smart, *Text-Book on Spherical Astronomy* (6th Edition, Revised by R. M. Green, Cambridge University Press, 1977), p. 40.

[3] Nautical Almanac Office,USNO, and Her Majesty's Nautical Almanac Office, UK Hydrographic Office, *Astronomical Almanac for the Year 2016,* p. B52.