

3D SCATTER PLOT OF HEART-SHAPED SURFACE

Roger L. Mansfield, January 31, 2011 (Second, Improved Version*)

Reference: <http://www.mathematische-basteleien.de/heart.htm>

$\varepsilon := 0.1$ Set tolerance for f-function.

Define functions to calculate points on surface of heart. ("Cor" is Latin for "heart.")

```
Cor(tol) := | m ← 0
            | for i ∈ 1..114
            |   z ← 1.0 ⋅ (-1 + (i-1)/50)
            |   for j ∈ 1..51
            |     y ← (j-1)/50
            |     n ← 0
            |     for k ∈ 1..201
            |       x ← 1.25 ⋅ (-1 + (k-1)/200)
            |       f ← (x2 + (9/4) ⋅ y2 + z2 - 1)3 - x2 ⋅ z3 - (9/80) ⋅ y2 ⋅ z3
            |       if |f| < tol
            |         | n ← n + 1
            |         | Fn ← |f|
            |         | x1n ← x
            |       if n ≥ 1
            |         | m ← m + 1
            |         | f ← min(F)
            |         | for p ∈ 1..n
            |           | Row ← (x1p y z) if Fp = f
            |         | H ← Row if m = 1
            |         | H ← stack(H, Row) otherwise
            | H
```

Heart1 := Cor(ϵ) (Heart1 is right front surface.)

$X1 := \text{Heart1}^{(1)}$ $Y1 := \text{Heart1}^{(2)}$ $Z1 := \text{Heart1}^{(3)}$

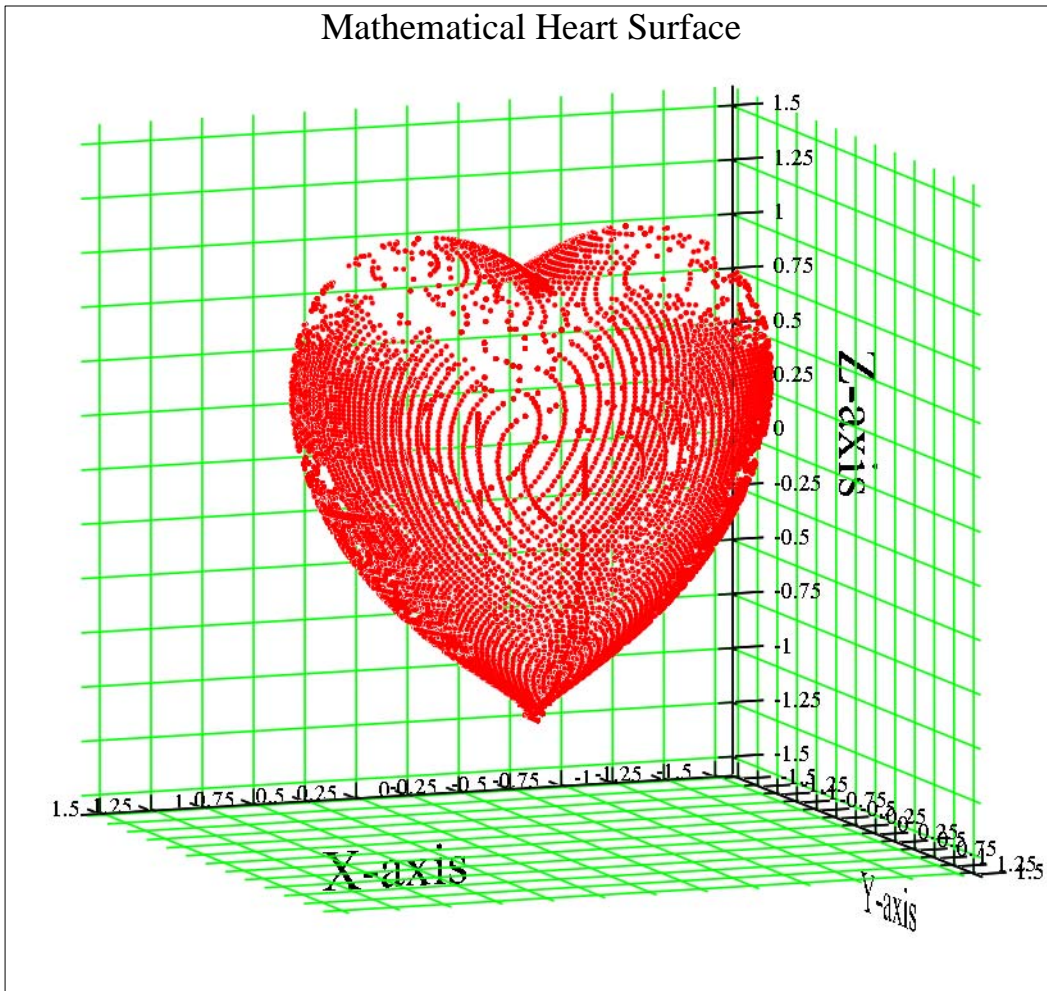
Heart2 := augment(augment($-X1, Y1$), $Z1$) (Heart2 is left front surface.)

$X2 := \text{Heart2}^{(1)}$ $Y2 := \text{Heart2}^{(2)}$ $Z2 := \text{Heart2}^{(3)}$

Heart3 := stack(Heart1, Heart2) (Heart3 is entire back surface.)

Heart3 := augment(augment($\text{Heart3}^{(1)}, -\text{Heart3}^{(2)}$), $\text{Heart3}^{(3)}$)

$X3 := \text{Heart3}^{(1)}$ $Y3 := \text{Heart3}^{(2)}$ $Z3 := \text{Heart3}^{(3)}$



$(X1, Y1, Z1), (X2, Y2, Z2), (X3, Y3, Z3)$

Notes on Construction

*This worksheet, Heart2-3D-MC15.xmcd, is an improved version of the Mathcad 13 worksheet Heart-3D-MC13-HP.xmcd that I posted to PlanetPTC on January 27, 2011. (See PlanetPTC Mathcad topic, "Mathematical ways to draw a heart".) This worksheet is shorter and faster.

It uses the first equation provided in the "Threedimensional" section of the Reference webpage, as opposed to the second equation, which is what the previous worksheet uses.

1. Basic Approach

The basic approach use herein is to calculate values of $f = f(x,y,z)$, with z ranging from -1.0 to 1.25, y ranging from 0.0 to 1.0, and x ranging from -1.25 to 0.0. Whenever the absolute value of f is less than the prescribed tolerance, the x -value is saved and a counter (n) is incremented.

When all of the x -values have been computed for a given z and y , then a row of the H matrix, Row = (x,y,z) is saved. The x -value is that x for which $|f|$ is the smallest. This ensures that of all the possible x values in an ε neighborhood of the true point on the heart's surface, the saved (x,y,z) point is the one closest to the true surface point. The H matrix contains all of the saved surface points; it is an $m \times 3$ matrix.

2. Implicit Function Theorem

The heart surface defines y implicitly as a function of z and x , piecewise on two half-surfaces. The first half-surface is the piece swept out by an xz plane starting at $y = 0.0$ and moving to $y = 1.0$ (this will be referred to as the "front" surface"). The second half-surface is the piece swept out by an xz plane starting at $y = 0.0$ and moving to $y = -1.0$ (this will be referred to as the "back surface"). The implicit function theorem assures that y is a single-valued function of z and x on each of the two half-surfaces.

3. Bilateral Symmetry

Function Cor is constructed so as to find at most one (x,y,z) on or near the heart surface as x sweeps from -1.25 to 0.0. But Cor only constructs the right half of the front surface. The left half of the front surface can be obtained by invoking the bilateral symmetry of the heart. That is, given the $m \times 3$ matrix Heart1 that describes the right side of the front surface of the heart, the left side of the front surface, Heart2, is just Heart1 with the first column, the column of x values, multiplied by -1.

4. Front-to-Back Symmetry

The xz plane ($y = 0$) cuts the heart into two equal halves, reminiscent of the two halves of a heart-shaped locket. Thus, invoking front-to-back symmetry, Heart3 is the mirror image of Heart1 and Heart2. It suffices then to plot Heart1, Heart2, and Heart3.